RADIATION TRANSFER IN A REAL SPECTRUM. INTEGRATION OVER FREQUENCY

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A new method for the calculation of heat exchange in systems with a complex spectrum containing spectral lines is worked out.

1. When calculating heat exchange by radiation in problems of thermal physics and gasdynamics, it is necessary to find values, integral with respect to the entire spectrum, of the flux or those of its divergences at any point of the volume of gas under consideration. As a rule, the problem is solved by successive approximations, when the profile of temperature and other parameters are assumed as given for finding the radiation field. After calculation of the radiation field the values of the radiation flux or those of its divergences are substituted into hydrodynamic equations, which allows the distribution of the parameters to be refined. Usually the iteration process converges fairly rapidly.

Fundamental difficulties arise in the calculation of the radiation characteristics. The radiation-transfer equations are formulated for spectral intensity, in connection with which their solution must be integrated over angles and frequency.

The radiation flux or its divergence entering into the equation of conservation of energy are quantities integral with respect to space, i.e., their values at a given point, generally speaking, depend on the distribution of the parameters in the entire volume of gas. Such a nonlocal character of the radiation field does not allow us, in the general case, to carry out exact integration of the spectral characteristics over frequencies and angles. In addition, direct integration over frequency of the radiation characteristics in the process of solution of the gasdynamic problem requires unrealistically much machine time. It is connected with the fact that hot gases usually have a complex spectrum of absorption and radiation, which are determinable by a large number of elementary photoprocesses. Particular difficulties are created by spectral lines in which the absorption coefficient often changes by several orders in a narrow frequency interval. These circumstances led to the development of approximate methods of solution of this problem.

In the present paper we briefly consider the methods of integration of the transfer equation over frequency, found in the literature, and propose a new method which solves the problem of calculation of selective radiation transfer in a number of cases.

All methods of integration of the transfer equation over frequency can be divided into five groups:

I. Limit approximations (homogeneous models, Planck's approximation, Rosseland approximation, partition of the spectrum according to optical density).

II. Local averaging (approximation of a "gray" gas, stepped models, group averaging).

III. The Milne–Eddington model.

IV. Nonlocal averaging (empirical models, homogeneous averaging, method of functionals).

V. Integral methods (method of equivalent width, band models, method of effective sections, integration along the outline, asymptotic method of partial characteristics). The first group contains methods not requiring the solution of the transfer equation over the entire volume of gas. Such a simplification is possible only in certain limiting cases. Thus, if the volume under consideration has virtually homogeneous distribution of temperature and pressure and possesses, in addition, a fairly high degree of symmetry, it can be com-

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pletely replaced by a homogeneous model volume, and it is possible to carry out beforehand the calculations of the heat-exchange characteristics. The most widely used models are homogeneous hemispheres and plane layers. Such calculations can basically serve for estimates.

In the limiting cases of small and large optical density, the Planck and Rosseland approximations are respectively used. The Planck approximation is usually valid only when the role of radiation is not large. Furthermore, it virtually may never describe radiation transfer in resonance lines and in an ionization continuum. For validity of the Rosseland approximation large optical densities, in the entire spectral region that is important in the energy relationship, are needed so that they usually are not encountered under terrestrial conditions. At the same time, the radiation is effectively reabsorbed and its role in the energy balance falls. Noncritical use of the Planck and Rosseland approximations, under conditions that do not satisfy those required for these approximations, can lead to an error in the order of magnitude and give qualitatively untrue results. In certain cases the spectrum of the absorption coefficient can be divided into portions with small and large optical density, while the role of portions with intermediate optical density is not large. Here, in individual regions of the spectrum, we can apply the Planck and Rosseland approximations.

The following three groups of methods presuppose a certain primary processing of the absorption coefficient in the real spectrum, in order to make the calculation of the radiation field feasible for computers.

In methods of local averaging, the absorption coefficient is averaged in fixed frequency intervals. The limiting case of such processing is approximation of a "gray" gas. As is shown in numerous papers, such an approximation does not usually reflect the qualitative side of the phenomenon and can give a considerable error. Various stepped models are basically applied for the description of radiation transfer in a continuous spectrum. An attempt to include spectral lines, having marked each line with a step, as was done by Olstad [1], was not crowned with success. Biberman [2] more than once pointed out that it is impossible, in principle, to replace a spectral line by a single equivalent step.

By the group averaging method [3] it is possible to describe the radiation transfer in spectral lines. In it a very detailed partition of the spectrum is carried out, so that each spectral line is replaced by a multistep model and the transfer in it is given by several average absorption coefficients. All spectral intervals with the same average absorption coefficient are joined in a single group, which is then characterized by a single transfer equation. The group averaging method leads to a small number of groups with a good description of the entire real spectrum; however, its use is confined to systems with comparatively small temperature variations, as, e.g., in compressed layers behind the front of shock waves [4]. With a large deviation of temperature from the temperature at which the partition of the spectrum was carried out, growth of the error is possible.

The use of the Milne-Eddington model, i.e., the assumption that the absorption coefficient can be represented in the form of the product of two functions, one of which depends on frequency, and the other on temperature and pressure, allowed a number of effective methods of averaging the transfer equation to be developed. To this group belong first and foremost the investigations by Gol'din and Chetverushkin [5], Shmyglevskii [6], and Krivtsov [7]. Regrettably, this model can seldom be applied to real spectra.

A series of attempts was made to broaden the frequency intervals in which averaging was carried out, by means of the use of the nonlocal character of the average absorption coefficient. This nonlocality manifests itself, e.g., in the fact that quanta emitted in a spectral line are absorbed nonuniformly along the ray. The quanta corresponding to the center of the line are absorbed more intensely, so that the principal contribution to the radiation intensity in the line is shifted as we move along the ray into the region of the wings.

Sampson [8] proposed an empirical combination of the coefficients of Planck, Rosseland, and optical density, which ensures an exit to the Planck and Rosseland coefficients in the case of an optically transparent and an optically dense medium. However, even for the simple spectrum of braking processes, the best combination found in [8] gave an error of nearly 100%. Another type of nonlocal averaging that does not use empirical combinations is proposed in [9]. The expression for the average nonlocal coefficient here is found from the condition that an exact solution is obtained for homogeneous volumes of gas. A check for a spectrum that was analogous to that used in [8] gave an error of about 5%; however, for a real spectrum with lines the error grew and reached 30% in certain cases.

An interesting variant of nonlocal averaging was proposed by Nemchinov [10]. The absorption coefficient in this method is averaged by means of the true intensity field, i.e., constitutes a functional of the radiation field. To compute such functionals we have first to solve the spectral transfer equation for all frequencies, after which it is possible to rapidly carry out a series of iterations using average coefficients. The method of functionals allows us to substantially reduce the machine time necessary for direct integration.

The methods enumerated can be successfully used for the calculation of the radiation transfer in simple spectra. For spectra containing a large number of spectral lines, integral methods turned out to be effective. In these earlier methods, before the solution of the problem, we calculate certain characteristics which are integral over parts of the spectrum or over the whole spectrum, and which then in the process of solution of the problem allow us to rapidly obtain the fields of intensities, fluxes, or their divergences. Integral methods have been used already for a long time in models of absorption bands — initially for isothermal layers and then, by means of the semiempirical approximation of Curtis—Godson, also for nonisothermal systems. In recent times, integral methods became to be used for real spectra. Simmons [11] proposed to use for nonisothermal layers an integral expression for the radiation intensity in a line with a dispersed outline. This expression, obtained by Ladenburg and Reiche for homogeneous layers, Simmons used in inhomogeneous gases by averaging the half-width of the line according to the Curtis—Godson method. Analogous computations for a line with a Doppler contour were carried out by Yamada [12]. These approximations were not widely used because of their unestablished accuracy.

An effective method to obtain integral expressions for the dispersion contour was worked out by Bronin and Lagar'kov [13]. It turned out that, having put the half-width in the denominator of the expression for the dispersion contour on the entire absorption path equal to the half-width at the source temperature, we can obtain very accurate results. This method of integration of the equation of radiation transfer is at the present time one of the best methods, but it is very complicated and accessible only to highly qualified specialists in the field of atomic physics. We should also mention the method of "effective sections" proposed by Ovsyannikov [14]. Here in the expressions for radiation flux and its divergences certain functionals are isolated which depend on the distribution of the parameters in the volume. In the general case these functionals cannot be tabulated beforehand. Only the case where the coefficient of absorption has the Milne—Eddington form is amenable to computations.

2. The approach of the present investigation consists of the fact that earlier, before the solution of the problem, for the entire real spectrum we carry out exact integration over the frequency, of the absorption capacity on the ray with a certain model distribution of the parameters [9, 15].

The radiation flux and its divergence in the general case can be calculated in terms of the intensity field:

$$\mathbf{S} = \int\limits_{(4\pi)} I \boldsymbol{\Omega} d\Omega, \tag{1}$$

$$\nabla \mathbf{S} = \int\limits_{(4\pi)} \nabla I d\Omega. \tag{2}$$

Here the intensity I and the quantity ∇I, which may be called the directed divergence, are determined by integration over the frequency, of the transfer equation for the spectral intensity

$$\boldsymbol{\Omega} \nabla I_\nu = k'_\nu (I^0_\nu - I_\nu). \tag{3}$$

For the ray shown in Fig. 1, the quantities I and ∇I at the point X are

$$I(X) = \int\limits_0^\infty \int\limits_0^L I^0_\nu(\xi) k'_\nu(\xi) \exp\left(-\left|\int\limits_\xi^X k'_\nu(\eta)\, d\eta\right|\right) \operatorname{sign}(X-\xi)\, d\xi d\nu, \tag{4}$$

$$\nabla I(X) = 2\int\limits_0^\infty I^0_\nu(X) k'_\nu(X)\, d\nu - \int\limits_0^\infty k'_\nu(X) \int\limits_0^L I^0_\nu(\xi) k'_\nu(\xi) \exp\left(-\left|\int\limits_\xi^X k'_\nu(\eta) d\eta\right|\right) d\xi d\nu. \tag{5}$$

For bounded and piecewise-continuous integrand functions we can alter the order of integration in expressions (4) and (5) and represent them in the form

$$I(X) = \int_0^L \Delta I(\xi,\ X)\,\mathrm{sign}\,(X - \xi)\,d\xi, \tag{6}$$

$$\nabla I(X) = 2\mathrm{So}(X) - \int_0^L \Delta\mathrm{Si}(\xi,\ X)\,d\xi, \tag{7}$$

where

$$\Delta I(\xi,\ X) = \int_0^\infty I_\nu^0(\xi)\,k_\nu'(\xi)\exp\left(-\left|\int_\xi^X k_\nu'(\eta)\,d\eta\right|\right)d\nu; \tag{8}$$

$$\mathrm{So}(X) = \int_0^\infty I_\nu^0(X)\,k_\nu'(X)\,d\nu; \tag{9}$$

$$\Delta\mathrm{Si}(\xi,\ X) = \int_0^\infty I_\nu^0(\xi)\,k_\nu'(\xi)\,k_\nu'(X)\exp\left(-\left|\int_\xi^X k_\nu'(\eta)\,d\eta\right|\right)d\nu. \tag{10}$$

The quantities ΔI, So and ΔSi can be called the partial intensity, the source, and the partial sink, respectively. The notation So and Si is formed from the words "source" and "sink."

Figure 2 clarifies the meaning of the term "partial." The intensity at the point X is made up of the partial intensities emitted at various points of the ray ξ and reaching as far as the point X with the absorption on the path $\xi \to X$ taken into account. If for the given partial intensity, having kept the temperature of the source and the length of the absorption path, we replace the actual temperature distribution by a certain model distribution, then we can first calculate the quantities (8)-(10) and then use them in the solution of heat exchange problems. In the role of the model distributions of temperature and pressure, it is convenient to use simple splines, i.e., piecewise-smooth functions. At the same time, quantities (8) and (10) will be functionals constructed with the use of these splines. In the process of solving the heat-exchange problem the actual profiles of temperature and pressure are approximated according to some rule by model splines, and from the block of partial characteristics we choose the quantities ΔI or ΔSi corresponding to these splines.

By increasing the order of the spline, we can attain an arbitrarily good approximation of the working profiles. In this sense the method is asymptotic and can give an arbitrarily high order of accuracy.

3. The efficiency of the asymptotic method is determined by the circumstance of how well it works already in the case of low orders of approximation, since for splines of high orders, although a high accuracy of the calculations is ensured, the number of approximation parameters can become unrealistically large. The effectiveness of the method being proposed is ensured by the fact that the absorption capacity of the ray does not depend on the actual distribution of the coefficient of absorption, but is determined only by the integral along the ray $\int k'_\nu(\eta)d\eta$. Calculations show that we can choose the approximating splines so that already for low orders of approximation (linear splines) a good accuracy of the calculations is assured.

In Fig. 3 we have shown the simplest two-temperature splines and three-temperature bilinear and quadratic splines (the dashed line) and methods of approximation by them of the working profiles of the temperature. In Fig. 3a, c and e the base temperatures T'_x and T'_η have been chosen according to a rule which will be discussed below.

Independently of the order of splines, the method gives accurate results in the limiting cases of low and high optical densities. For low optical density the exponents in expressions (8) and (10) become unity, the integral term in (7) ceases to play an important part, and calculations according to expressions (6) and (7) give accurate results for an optically transparent gas. For high optical density the part of the ray playing a part in the formation of the intensity at the point X (see Fig. 1) becomes small, so that the distribution of tempera-

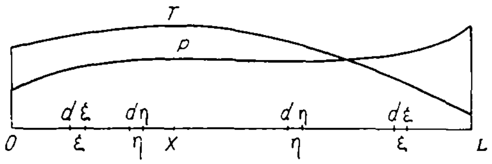

Fig. 1. Ray of integration with an arbitrary distribution of temperature T and pressure P.

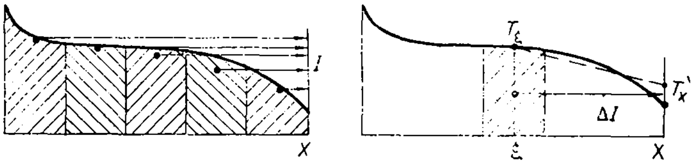

Fig. 2. Explanation of the meaning of partial characteristics. Radiating elements are hatched. The dashed line denotes approximation of the temperature distribution on the absorption path by linear splines.

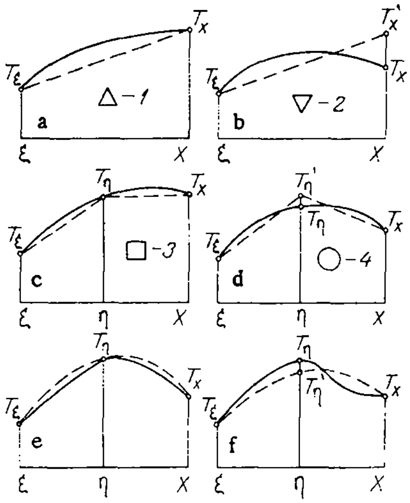

Fig. 3. Approximation of the working profiles of the temperature by linear (a, b), bilinear (c, d), and quadratic (e, f) splines: 1-4) see Fig. 7.

ture and pressure on this part approaches a linear distribution. Approximation of such parts by any splines is accurate, and this in fact ensures that an accurate solution is obtained. Thus, only the case of an intermediate optical density need be subjected to a check.

4. A model spectrum, constituting a schematized spectrum of atomic hydrogen, was used for a check. The braking processes of electrons in proton fields, photoionization from the basic state and with the excited levels and series of Lyman and Balmer were taken into account. The absorption coefficient of the model spectrum for a number of temperatures and pressures is presented in [9, 15].

In the role of the partial characteristic for the calculation of the intensity field, expression (8) is used, in which the integral under the exponent sign is a functional of splines modeling the temperature and pressure distribution. Thus, for linear splines $\Delta I = \Delta I(T_\xi, P_\xi, T_X, P_X, x)$, where $x = |\xi - X|$. The final expression for the calculation of ΔI has the form

$$\Delta I = \int_0^\infty I_\nu^0(T_\xi)\, k'_\nu(T_\xi,\ P_\xi) \exp\left(-\int_0^x k'_\nu(\eta)\, d\eta\right) d\nu. \tag{11}$$

For the calculation of ∇I expressions (9) and (10) are often inconvenient. The presence of portions with a large absorption coefficient (spectral lines, photoionization from the basic state) in the spectrum leads to the quantity ∇I in (7) being orders less than the quantities on the right-hand side, as a result of which computation with a very fine step with respect to ξ is required. Expressions free from this shortcoming are obtained from (9) and (10) by means of the identity transformation:

$$\mathrm{Som} = \int_0^\infty I_\nu^0(T_X)\, k'_\nu(T_X,\ P_X) \exp\left(-\int_0^x k'_\nu(\eta)\, d\eta\right) d\nu, \tag{12}$$

$$\Delta\mathrm{Sim} = \int_0^\infty [I_\nu^0(T_\xi) - I_\nu^0(T_X)]\, k'_\nu(T_\xi,\ P_\xi)\, k'_\nu(T_X,\ P_X) \exp\left(-\int_0^x k'_\nu(\eta)\, d\eta\right) d\nu. \tag{13}$$

In expressions (12) and (13), which have been modernized, the notation So and Si are provided with the additional letter m. The functionals Som and ΔSim depend on the same variables as ΔI. In addition, we see that the new source ΔI coincides (with accuracy up to the indices) with the quantity ΔI; therefore, it is preliminarily necessary to calculate only two functionals: ΔI and ΔSim.

The computation of ∇I by means of expressions (12) and (13) is carried out according to the expression

$$\nabla I(X) = \mathrm{Som}\Big|_0^X + \mathrm{Som}\Big|_X^L - \int_0^L \Delta\mathrm{Sim}\, d\xi. \tag{14}$$

The approximation of the working profiles by splines with the use, in the role of the base points, of the temperatures and pressures of these profiles (Fig. 3a, c, and e) is the simplest, but it does not realize all the possibilities of splines of the given order. The accuracy can be substantially increased if we choose the temperatures and pressures T_X' and P_X' in the approximation according to the method shown in Fig. 3b or T'_η, P'_η according to Fig. 3d, f from the condition of conservation of the integrals of temperature and pressure. Here the calculation expressions, e.g., for the temperatures T_X' and T'_η in the case of linear and bilinear splines have the form

$$T'_X = 2\int_\xi^X \frac{T(\eta)\, d\eta}{|X-\xi|} - T_\xi, \tag{15}$$

$$T'_\eta = 2\int_\xi^X \frac{T(\eta)\, d\eta}{|X-\xi|} - \frac{T_\xi + T_X}{2}. \tag{16}$$

The approximation of the working profiles by linear splines according to this method is shown in Fig. 2 by the dashed line.

The approximation method just presented is based on the circumstance that in the case of linear dependence of the absorption coefficient on temperature and pressure the optical density does not depend on the concrete distribution of the parameters, but is determined only by their integral. Thus, if $k'_\nu(T) = A + BT$, then $\int_0^X k'_\nu\, d\eta = Ax + B\int_0^X T d\eta$. Linearity of the absorption coefficient is always observed for small $|\xi - X|$. (In the general case, the rule just presented will be approximate and needs checking.) In Fig. 4 we have shown the behavior of the absorption coefficient k_ν' on the ray dependent on the temperature, for three temperature

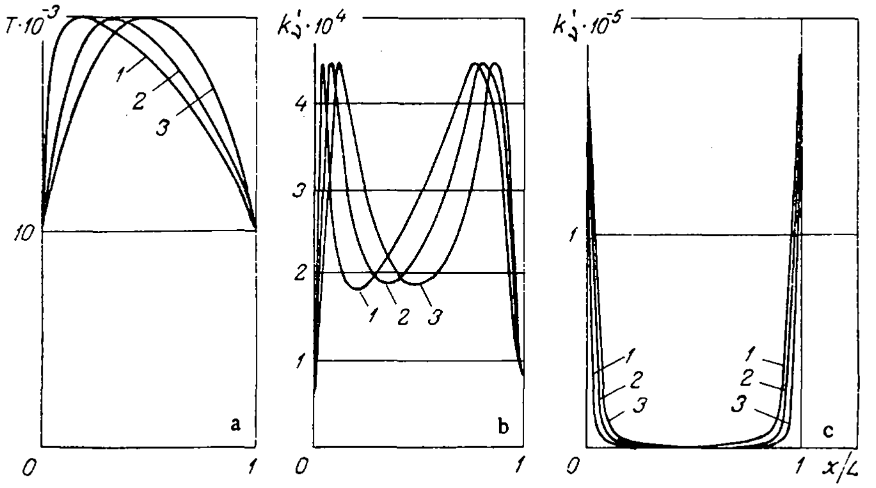

Fig. 4. Variation of the absorption coefficient along the ray with different temperature distribution (a). The numbers at the curves correspond to the numbers of the temperature profiles; b) region of continuous spectrum, $\nu = 5\cdot10^4$ cm^{-1}; c) center of line L_α.

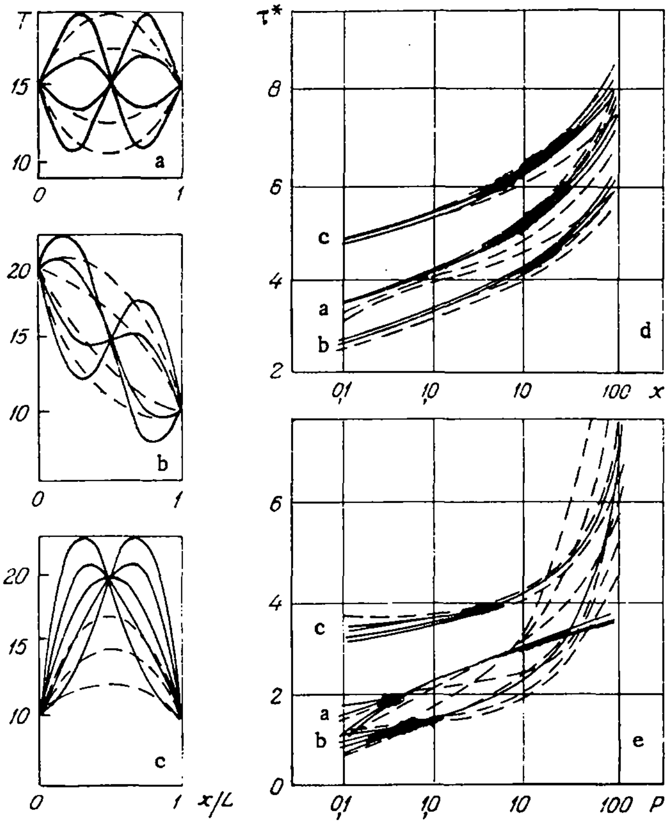

Fig. 5. Effective optical density for different temperature profiles, T, $10^3\cdot°K$; x, cm; P, bar. Solid curves) in d and e correspond to the solid curves in a, b, and c which retain the integral $\int T d\eta$; d) P = 10 bar; e) L = 10 cm.

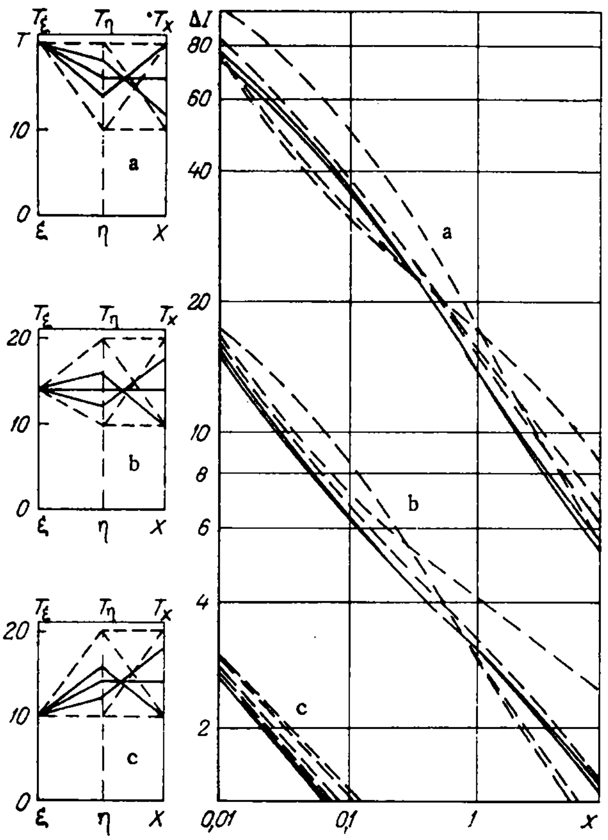

Fig. 6. Partial intensities for bilinear temperature splines. x, cm; ΔI, arbitrary units.

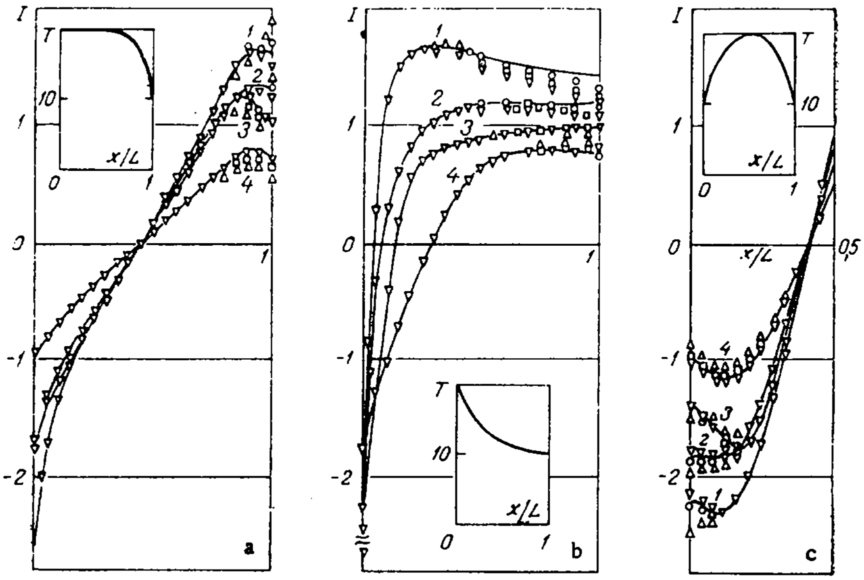

Fig. 7. Verification of the method of partial characteristics for a series of temperature profiles. T, $10^3 \cdot {}^\circ K$, I; relative units: a, c) P = 10 bar; 1) L = 100 cm; 2) 10; 3) 1; 4) 0.1; b) L = 10 cm; 1) P = 100 bar; 2) 10; 3) 1; 4) 0.1.

profiles with the same integral $\int T d\eta$(1, 2, 3). Case b corresponds to the portion of continuous spectrum and c to the resonance line L_α. We see that the integral $\int k'_\nu\, d\eta$ is approximately conserved.

5. The verification of the method was carried out for numerous temperature and pressure profiles, including typical profiles encountered in experimental and technological systems. The temperature, pressure, and dimensions of the radiating volumes were widely varied.

In Fig. 5, we have shown for a number of temperature distributions the behavior of the quantity

$$\tau^* = \ln(\Delta I(T_\xi, T_X, 0)/\Delta I(T_\xi, T_X, x)), \tag{17}$$

having the meaning of an effective optical density. By solid lines we have shown the temperature splines which conserve the integral $\int T d\eta$, and the values of τ^* corresponding to these splines. We see that the rule is well observed. It should be noted that analogous calculations carried out for portions of a model spectrum showed a stronger dependence of τ^* on the temperature distribution, a circumstance which speaks for a certain compensation of errors when calculating for the entire spectrum. The dashed curves are the remaining temperature profiles (i.e., not preserving $\int T d\eta$) and the values of τ^* and ΔI corresponding to these profiles.

In Fig. 6 we have shown the values of partial intensity for bilinear splines. The meaning of the solid lines and dashes is the same as in Fig. 5. We note that the weak dependence of ΔI on the temperature distribution under the condition of conservation of the integral $\int T d\eta$, which is observable from Fig. 6, already by itself guarantees high accuracy of the method being proposed. Nevertheless, the method was subjected to a comprehensive and careful check. In Figs. 7-9 we have shown certain results. The solid lines are the results of direct integration over frequency by means of expressions (4) and (5). Here (5) or the expressions (7), (9), and (10) equivalent to it were transformed in the same way as (12) and (13).

The results of the calculations by means of the method of partial characteristics are shown in Figs. 7-9 by means of marks 1-4 corresponding to the method of approximation of the working profiles by splines (see Fig. 3a-d). In Fig. 9 the marks at the curves correspond to the approximation method of the expression for a sink (13). Source (12) was always approximated according to the method of Fig. 3b. Analogous results were obtained for other temperature and pressure profiles. It should be noted that the values of ∇I necessary for the calculation of $\nabla \mathbf{S}$ are calculated somewhat worse than I. This result, being a natural consequence of the circumstance that ∇I in fact is a derivative of I, does not diminish the accuracy of the final solution of the heat-exchange problem. The integral results of the solution will have an accuracy which is analogous to the accuracy of calculation of the radiation flux **S**, and not that of its divergence $\nabla \mathbf{S}$ which enters into the differential equation of energy conservation.

6. The method just presented does not require introduction of any assumptions about the form of the spectrum. Any number of spectral lines with an arbitrarily complex contour, with multistrandedness and shift taken into account, can be included in the calculation. The results of the calculations do not depend on the form of the spectrum and, for a spectrum of any complexity, constitute smooth relationships which are easily entered into the computer memory.

An important advantage of the asymptotic integral method is the fact that it allows us to carry out in many cases the computation on a gasdynamic grid the step of which depends only on the temperature and pressure gradients. The traditional methods, as a rule, require for the calculation of the radiation transfer introduction of special grids, where the step depends on optical density.

The method worked out here is accessible to wide circles of thermal physicists and gasdynamicists who have no special preparation in the field of atomic spectroscopy. The calculation of the radiation flux or its divergence reduces here to approxmation of the working profiles of temperature and pressure by model splines, selection from the block of the characteristics those corresponding to these splines, and summation of them. Here the computation time becomes small in comparison with the time of solution of the gasdynamic problem. The calculation of the radiation transfer by the traditional methods usually results in an inverse relation of the times.

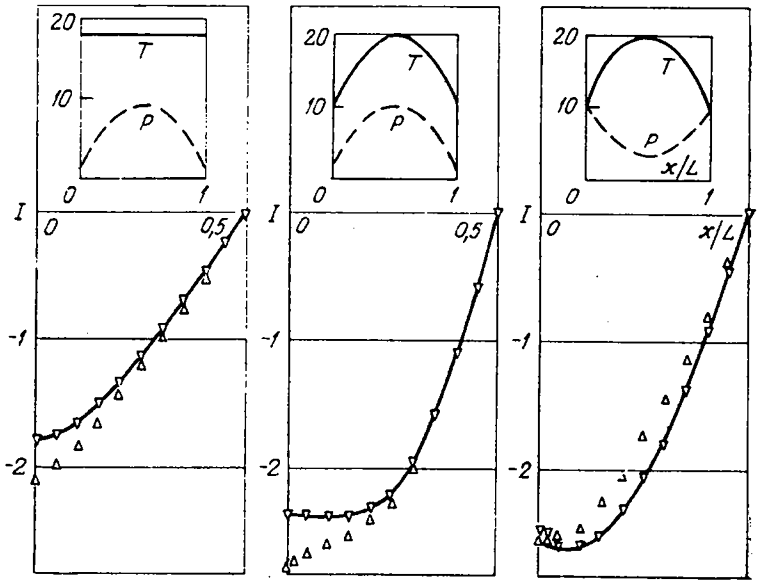

Fig. 8. Verification of the method of partial characteristics. T, $10^3 \cdot °K$; P, bar; I, relative units.

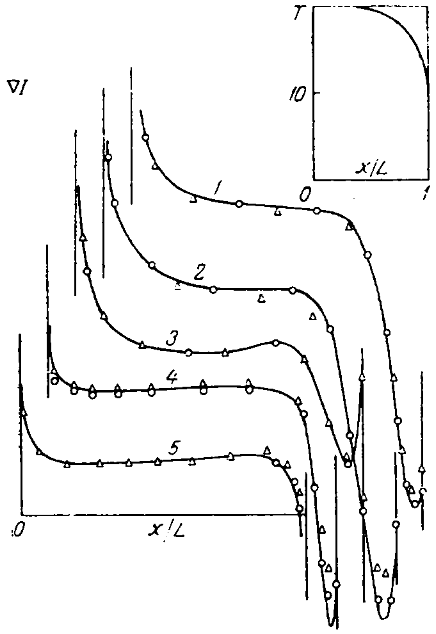

Fig. 9. Verification of the method of partial characteristics. T, $10^3 \cdot °K$; ∇I, units: 1) L = 0.1 cm, P = 10 bar; 2) respectively, 1 and 10; 3) 10 and 10; 4) 10 and 1; 5) 10 and 0.1.

NOTATION

X, x, ξ, η, coordinates; ν, frequency; P, pressure; T, temperature; k'_ν, absorption coefficient with forced emission taken into account; I_ν, spectral intensity; I^0_ν, spectral equilibrium intensity; I, frequency integral intensity; τ, optical density; Ω, unit vector; S, radiation flux; A, B, constants.

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MODELING RADIANT-HEAT-TRANSFER PROBLEMS IN MEDIA OF NONPLANE GEOMETRY

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Approximate and numerical methods of solution of radiation-transfer equations in cylindrical and spherical media are proposed, the spectroscopic luminescence characteristics of infinite and finite cylinders are analyzed, and an algorithm for their calculation is given.

In many problems of radiant heat transfer, it is necessary to take account of multiple-scattering processes, since the heat carrier in various power stations is a two-phase gas–solid-particle system. The investigation of multiple-scattering laws is also of great importance for other fields of physics and for physical-engineering applications (atmospheric optics, the energetics of planetary atmospheres, the problem of spacecraft entry into the atmospheric layer, the interaction of laser radiation with matter, etc.). The intensification of modern power stations, associated with the considerable increase in heat-carrier temperature, requires as accurate as possible a determination of their thermodynamic characteristics. At the same time, the radiant component in the total energy balance becomes significant, and therefore the correct solution of radiant-heat-transfer problems is a pressing concern. On the one hand, it is necessary to establish the basis of radiation-transfer equations for real physical models and the limits of applicability of this solution; on the other, it is necessary to use reliable spectroscopic characteristics of the media investigated.

The problem of radiation propagation in two-phase media of nonplane geometry is one of the most important in modern radiation-transfer theory. Because of the great mathematical difficulties involved, approximate [1-3] or numerical [4, 5] methods are usually used for the solution of integrodifferential radiation-transfer equations. Note that the development of approximate methods of solution is expedient both for rapid estimates of the energy characteristics of two-phase nonplane media and for the determination of the best initial approximation in numerical calculations of radiation-transfer equations by iterational methods. The wide use of computers allows numerical experiments to be carried out for diverse physical phenomena, which, in economic terms, is considerably preferable to full-scale experiments and physical modeling. By constructing mathematical models, it is possible to study the important physical laws governing phenomena or to investigate directly conditions of power-

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